THAMMASAT REVIEW
PRINT ISSN: 0859-5747 ONLINE ISSN: 2630-0303

Volume 27 No 2 (July-December) 2024 [Page 32-51]

Innovative Multicurrency Portfolio Optimization Using Copula-Based Scenarios

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Received 2 March 2024; Received in revised form 16 July 2024
Accepted 29 July 2024; Available online 24 November 2024

Abstract

Optimizing a multicurrency portfolio requires a flexible model to manage exchange rate risk as well as representational data on asset-currency dependency. Additionally, deliberate scenario generation is also vital for portfolio risk evaluation, especially for the case of extreme events. This study proposes a mean-CVaR portfolio optimization model with currency overlay under regular-vine copula generated scenarios. To highlight the importance of the scenario generation technique, the performance of the resulting portfolios from the proposed method are compared with those optimized under multivariate normal assumption. The results show that portfolios from our proposed approach outperform those from the traditional method, both in return and risk metrics. This outperformance is largely attributed to active currency hedging, which takes advantage of detailed information captured by a regular-vine copula.

Keywords

International portfolio, Currency overlay, Regular-vine copula

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Introduction

Markowitz's innovative work in portfolio optimization, as outlined in Markowitz (1952), initially tackled risk minimization by targeting the variance of returns while aiming for a specified level of expected return. Subsequent iterations of portfolio optimization problems have replaced variance with Conditional Value-at-Risk (CVaR) to better address downside risk. Rockafellar and Uryasev's contribution, as discussed in Rockafellar et al. (2000) introduced the mean-CVaR portfolio implementation, which has since become a standard approach among practitioners. However, the effectiveness of the CVaR risk measure is heavily reliant on accurately modeling the distribution of portfolio returns.

Traditional methods in finance often characterize probability distributions using the first four statistical moments: mean, variance, skewness, and kurtosis. Dependencies between distributions are typically represented using correlation matrices, as discussed in Høyland et al. (2003) and Kaut et al. (2007). However, these methods are limited in their ability to describe the intricate relationships between asset returns, particularly when dealing with outliers or non-linear dependencies. Correlation measures, such as Pearson's correlation coefficient Pearson (1895), capture only linear relationships, which can be inadequate for modeling the complex behavior of financial returns. Rank-based correlations, like Kendall's tau (Kendall, 1938), offer a more robust alternative for capturing non-linear dependencies.

In reality, financial asset returns exhibit non-Gaussian behavior with asymmetric dependence structures. Empirical evidence indicates that returns are more strongly correlated during market downturns compared to periods of market stability or growth (see, for instance, Ang & Bekaert, 2002; Ang & Chen, 2002; Campbell et al., 2002; Erb et al., 1994; Longin & Solnik, 2001; Mitchell & Pulvino, 2001; Patton, 2004). This phenomenon highlights the inadequacy of models assuming normality and linear dependence in accurately reflecting market dynamics. To address these limitations, copulas have been introduced as a more versatile tool for modeling the dependence structure of asset returns.

The concept of copulas, introduced by Sklar (1959), provides a method to construct joint distributions by linking marginal distributions. This approach, further elaborated by Nelsen (2007), allows for the independent modeling of marginal distributions and their dependence structure. Unlike the multivariate normal distribution, which assumes Gaussian marginals with linear dependencies, copulas enable the combination of marginal distributions from various families, thus accommodating non-normal characteristics such as heavy tails and asymmetric dependencies. This flexibility significantly enhances the robustness of risk management models. Despite the advantages of copulas, previous studies (Kaut, 2014; Kaut & Wallace, 2011; Sutiene & Pranevicius, 2007) have shown that empirical copulas may be unreliable with small sample sizes and that single-family copulas used in multivariate models limit flexibility in high dimensional settings. To overcome these challenges, this study adopts vine copulas (Bedford & Cooke, 2002), which decompose high-dimensional dependence structures into a series of bivariate copulas organized hierarchically. This method, described in Kielmann et al. (2022) and Karakaş (2022), provides greater flexibility and accuracy in modeling complex dependencies.

The specific issue addressed in this study is the inadequacy of traditional multivariate normal distribution models in capturing the complex, non-linear dependencies and asymmetric behaviors of asset returns, especially during market downturns. Current literature has largely focused on models assuming normality and linear dependence, which fail to account for the extreme events and tail dependencies observed in financial markets. This gap highlights the need for more robust modeling techniques that can better represent the true nature of asset return distributions. This paper applies the vine-copula model to real-world data, focusing on the optimization of international portfolios using a mean-CVaR framework. It introduces a novel multi-currency portfolio optimization model that incorporates currency overlay through foreign exchange forwards to manage currency exposure. The optimization model accounts for transaction costs and hedging expenses associated with exchange rate fluctuations, as detailed in Chatsanga and Parkes (2017).

The research question guiding this study is: How can a copula-based model be developed to improve the accuracy and robustness of multicurrency portfolio optimization under various market conditions? This study contributes to the existing literature by demonstrating the application of vine copulas in capturing complex dependencies among asset returns and by comparing the performance of these models against traditional multivariate normal distributions. Our findings reveal that vine copula-based models provide superior risk-adjusted returns and better manage extreme market events, thereby offering a significant advancement in portfolio optimization techniques. Additionally, the economic context for this study involves significant market volatilities and varying economic conditions of the selected currencies (USD, GBP, EUR, JPY and CNY). These currencies were chosen due to their substantial influence on global financial markets and the diversity they bring in terms of economic environments. Historical data on these currencies reveal fluctuations influenced by geopolitical events, monetary policies, and market sentiment. Our model integrates these factors, ensuring that the scenarios generated reflect realistic market conditions, thereby enhancing the practical applicability of our findings.

In summary, this study aims to fill a notable gap in the literature by providing a comprehensive and flexible model for multicurrency portfolio optimization. By leveraging the strengths of vine copulas, we offer a method that not only improves the representation of dependencies among asset returns but also enhances the overall risk management strategy in international investments. The findings of this research have significant implications for both academic research and practical portfolio management, offering a robust tool for navigating the complexities of global financial markets.

The structure of the paper is organized as follows: Section 2 presents the approach using regular-vine copulas for scenario generation, construction of currency overlay, and formulation of the optimization model. Section 3 presents the experimental results with analyses, and Section 4 concludes the study.

Methodology

This section presents the comprehensive methodology employed in our study to optimize a multi-currency portfolio using regular-vine copulas and the mean-CVaR framework. We begin by describing the scenario generation process, utilizing empirical marginal distributions and regular-vine copulas to capture nonlinear dependencies among asset returns. Next, we detail the formulation of the multi-currency portfolio optimization problem, incorporating currency overlay techniques to manage exchange rate risk. The methodology also includes the estimation of empirical distributions, the construction of joint distributions, and the generation of multiple scenarios.

We employ Conditional Value-at-Risk (CVaR) as the risk measure in this study due to its ability to provide a more comprehensive assessment of tail risk compared to traditional measures like volatility and Value-at-Risk (VaR). While VaR indicates the maximum potential loss at a certain confidence level, it does not account for the magnitude of losses beyond this threshold. In contrast, CVaR captures the expected losses occurring in the tail of the loss distribution, offering a clearer picture of extreme risk events. This characteristic is particularly valuable in portfolio optimization, as it allows for better risk management under adverse market conditions. Additionally, CVaR is coherent, satisfying properties such as subadditivity and convexity, which are desirable for constructing diversified portfolios. These features make CVaR a robust choice for managing risks in a multi-currency portfolio.

Finally, we outline the evaluation of the optimized portfolios through backtesting and the decision-making processes involved in selecting the optimal portfolio. This detailed approach ensures the robustness and applicability of our model to real-world financial markets.

Creating Scenarios with Regular-Vine Copulas.

The paper outlines a method for scenario generation in our optimization problem. In essence, we employ empirical marginal distributions to avoid assumptions about asset return distributions. To address nonlinear dependencies, we construct a joint distribution using a regular vine copula.

1) Modelling marginal distributions - We model marginal distributions by fitting an invertible empirical distribution to each financial return time series and estimating a marginal probability distribution function (PDF) based on empirical data. We use kernel density estimation (KDE) to estimate an empirical PDF of a return time series. For a random variable Ξ_i with m independent observations $\xi_{i1}, \dots, \xi_{im}$, the kernel density estimator approximates the density value at a point *x* in the PDF as follows:

$$f(\xi_i) = \frac{1}{mh} \sum_{j=1}^{m} K\left(\frac{\xi_{ij} - \xi_i}{h}\right). \quad (1)$$

In our analysis, we use the Epanechnikov kernel as the kernel function denoted by K, and we determine an optimal bandwidth h following Silverman's rule of thumb (Silverman, 1986). We then create an empirical cumulative distribution function (CDF) for each return series based on the estimated PDF as follows:

$$F(\xi_i) = \sum_{\xi_{ij} \le \xi_i} f(\xi_{ij}). \quad (2)$$

The resulting CDF is uniform within the range $[0,1]$ and serves as an input parameter for a copula function. In the context that follows, we represent the CDF of a random variable i as u_i.

2) Estimating a regular-vine copula - To fit an R-Vine copula to a given dataset, Dissmann et al. (2012) outline the procedure as follows:

(a) Choose the R-Vine structure by determining the unconditioned and conditioned pairs for pair-copula construction.

(b) Fit a pair-copula family to each selected pair in step (a).

(c) Estimate the parameters corresponding to each copula.

In our research, we employ the sequential method introduced in Dissmann (2010), which involves fitting an R-Vine tree-by-tree approach, to estimate the R-Vine copula. The VineCopula package in R (Schepsmeier, 2012) is used to estimate the R-Vine copula model. This step yields the optimal combination of bivariate copulas and conditional bivariate copulas within the R-Vine structure for the dataset available.

3) Sampling from a regular-vine density - We adopt the R-Vine sampling approach described in [24]. This process begins by sampling $u_1, \dots, u_n$ which are independent and uniformly distributed on the interval $[0,1]$. Then set:

$$\begin{aligned}
\xi_1 &= u_1, \\
\xi_2 &= F_{2|1}^{-1}(u_2|\xi_1), \\
\xi_3 &= F_{3|12}^{-1}(u_3|\xi_1, \xi_2), \\
&\vdots \\
\xi_n &= F_{n|12\dots n-1}^{-1}(u_n|\xi_1, \dots, \xi_{n-1}),
\end{aligned} \tag{3}$$

where $F_{j|12\dots j-1}^{-1}(u_n|\xi_1, \dots, \xi_{j-1})$ for $j = 1, \dots, n$ represents the inverse of the conditional cumulative distribution function introduced by Joe (1996). By solving a set of equations (3), we obtain dependent $\xi_1, \dots, \xi_n$, forming a collection of single scenarios for *n* random variables. To generate *N* scenarios, the process of randomly sampling $u_1, \dots, u_n$ is repeated N times.

Formulating a Multi-Currency Portfolio Optimization Problem.

In an international portfolio, alongside market risk associated with asset returns, there is also exposure to exchange rate risk. To address this risk, investors often employ currency overlay techniques, which involve adjusting currency exposure using exchange rate derivatives to either speculate or hedge against exchange rate fluctuations based on their preferences. The optimization problem formulation presented subsequently is adapted from the approach outlined by Chatsanga and Parkes (2017).

Selection of Currencies and Home Countries.

The currencies selected for this example are the US Dollar (USD), Euro (EUR), and Japanese Yen (JPY). These currencies were chosen based on their substantial influence on global financial markets and the diversity they represent in terms of economic environments. The United States, the Eurozone, and Japan are among the largest and most economically significant regions globally[1], making their currencies highly relevant for international portfolio optimization. In what follows, the portfolio funding currency is chosen as USD.

2.2.2. *Portfolio Structure with Overlay Constraints.* When constructing a portfolio that invests in multiple countries, there are two primary sources of returns that impact the portfolio's overall market value. The first source stems from asset prices along with dividends or other forms of interest-bearing income, while the second source relates to currency fluctuations leading to gains or losses. Consequently, each country's investment within the portfolio reflects a combination of exposure to asset markets and exposure to currency exchange rates. This setup also allows for the adjustment of currency exposure, thereby mitigating risky

[1] Based on Statista Search Department (May 21st, 2024) Triennial forex daily volume with 39 different currencies 2001-2022. Statista. https://www.statista.com/statistics/247328/ activity-per-trading-day-on-the-global-currency-market/.

foreign currency positions. Currency overlay refers to altering currency exposure, which influences the initial currency holdings of an unhedged portfolio.

Table 1 Sample portfolios with and without currency overlay. An overlay position is determined by the deviation of currency exposure from asset exposure. The total overlay is calculated as half of the absolute sum of all overlay positions. The portfolio funding currency is USD.

	Hedged			Unhedged		
	US	UK	JP	US	UK	JP
asset exposure (%)	35	45	20	35	45	20
currency exposure (%)	27	52	21	35	45	20
overlay position (%)	-8	7	1		-	
total overlay (%)		8			-	

A currency overlay comprises overlay positions, illustrated in Table 1, which arise from holding one or more foreign exchange forward contracts (FX forwards) as shown in Table 2. Each FX forward contract incurs a "cost of carry" or hedging cost, which can be positive or negative depending on the interest rate differential between the currency pairs. For instance, consider a portfolio incorporating three FX forwards outlined in Table 2. The cost of carry for each forward contract depends on the currencies exchanged, the corresponding interest rates, and the position within the portfolio. Selling JPY for USD at 2% of the portfolio size results in a positive carry of $2\% \times 2\% + 1\% \times (-2\%) = 0.02\%$ for the portfolio. Conversely, selling GBP for JPY leads to a negative carry of $4\% \times (-3\%) + 1\% \times 3\% = -0.09\%$ due to shifting exposure from a high-interest-rate country to a low-interest-rate one. The total overlay position constitutes 8% of the portfolio, with a positive carry of 0.13% from the combined three forward contracts. This carry amount is then added to the overall portfolio return.

Hence, the net cost of carry is the combined total of interest rates and overlay positions. In the case of an investment in any country j, the overall contribution to the portfolio's total return is as follows:

$$r_j = a_j r_j^a + c_j r_j^c + v_j i_j, \tag{4}$$

where r_j represents the total return generated from investing in country j, while a_j, c_j, and v_j denote the asset exposure, currency exposure, and overlay position related to country j, respectively. Additionally, r_j^a, r_j^c, and i_j stand for the expected asset return, expected currency return, and expected interest rate associated with country j.

Given that an overlay position is defined as the deviation of currency from asset exposures, equation (4) can be alternatively represented as:

$$r_j = a_j r_j^a + c_j r_j^c + (c_j - a_j) i_j \quad (5)$$
$$= a_j (r_j^a - i_j) + c_j (r_j^c + i_j).$$

Table 2 The cost of carry pertains to the expenses associated with forward contracts on foreign exchange rates. The figures in bold represent portfolio positions, stated as percentages. The total currency overlay position for each currency is determined by aggregating the net forward positions related to that currency. The cost of carry for holding each forward contract is calculated as the weighted sum of interest rates and forward positions concerning the currencies involved in the forward contract. The portfolio funding currency is USD.

	USD	GBP	JPY	Cost of Carry
interest rate (%)	2	4	1	
sell JPY, buy USD (%)	**2**		**-2**	0.02
sell USD, buy GBP (%)	**-10**	**10**		0.20
sell GBP, buy JPY (%)		**-3**	**3**	-0.09
overlay (%)	**-8**	**7**	**1**	0.13

We designate $r_j^a - i_j$ and $r_j^c + i_j$ as the adjusted returns for assets and currencies, respectively. Equation (5) illustrates that the total return of the portfolio, including returns from assets, currencies, and foreign exchange forward carry costs, is the product of adjusted returns, asset exposure, and currency exposure. This indicates that the computation of overlay positions is unnecessary for determining a portfolio's total return. Moreover, in scenarios where a portfolio does not involve forward contracts, the interest rate terms in equation (5) cancel out, highlighting that the formulation presented in equation (5) offers a generalized method for calculating total returns in international portfolios.

To construct a portfolio with currency overlay, let $\mathbf{f}_k = (f_{k1}, \dots, f_{kC})$ be a vector representing exposure from a forward contract k, where $K = \binom{C}{2}$ denotes the total number of forward contracts available for investment across *C* countries. The specification of a forward contract dictates that only two elements of $\mathbf{f}_k$ signify the exposure, with one being the negative value of the other, while the remaining elements are zero. To simplify the constraints in an optimization problem, we define f_{kj} as an element within a matrix **F**, wherein:

$$\mathbf{F} \overset{\text{def}}{=} \mathbf{T} \circ (\mathbf{1}^T \otimes \mathbf{q}) \tag{6}$$

where $\circ$ represents the Hadamard product operator, while $\otimes$ denotes the Kronecker product operator. **T** is a combinatorial matrix of size $K \times C$ with entries from the set $\{-1,0,1\}$. **1** is a column vector of ones with dimensions $C \times 1$, and **q** is a column vector of size $K \times 1$ that determines the size of exposure. Further elaboration on the formulation discussed above can be found in Chatsanga and Parkes (2017).

The Portfolio Optimization Problem with Currency Overlay.

Using the currency overlay portfolio structure detailed in Table 3, we can establish the portfolio optimization problem using the following symbols and notations:

a : A vector of asset exposure; $\boldsymbol{a} = [a_{11}, \ldots, a_{ij}, \ldots, a_{AC}]^T$.

c : A vector of currency exposure; $\boldsymbol{c} = [c_1, \ldots, c_j, \ldots, c_C]^T$ where $c_j = \sum_{i=1}^{A} a_{ij} + \sum_{k=1}^{K} f_{kj}\ ;\ j = 1, \ldots, C$.

x : A vector of decision variables; $\boldsymbol{x} = [\boldsymbol{a}, \boldsymbol{c}]^T$.

r : A vector of adjusted returns; $\boldsymbol{r} \in \mathcal{R}^{C(A+1)}$.

μ : The target return of a portfolio.

Table 3 Structure of an international portfolio with currency overlay

	Country 1	⋯	Country j	⋯	Country C
Asset class 1	a_{11}	⋯	a_{1j}	⋯	a_{1C}
⋮	⋮	⋱	⋮	⋱	⋮
Asset class i	a_{i1}	⋯	a_{ij}	⋯	a_{iC}
⋮	⋮	⋱	⋮	⋱	⋮
Asset class A	a_{A1}	⋯	a_{Aj}	⋯	a_{AC}
Forward position 1	f_{11}	⋯	f_{1j}	⋯	f_{1C}
⋮	⋮	⋱	⋮	⋱	⋮
Forward position k	f_{k1}	⋯	f_{kj}	⋯	f_{kC}
⋮	⋮	⋱	⋮	⋱	⋮
Forward position K	f_{K1}	⋯	f_{Kj}	⋯	f_{KC}
Asset exposure	$\sum_{i=1}^{A} a_{i1}$	⋯	$\sum_{i=1}^{A} a_{ij}$	⋯	$\sum_{i=1}^{A} a_{iC}$
Overlay position	$\sum_{k=1}^{K} f_{k1}$	⋯	$\sum_{k=1}^{K} f_{kj}$	⋯	$\sum_{k=1}^{K} f_{kC}$
Currency exposure	$\sum_{i=1}^{A} a_{i1} + \sum_{k=1}^{K} f_{k1}$		$\sum_{i=1}^{A} a_{ij} + \sum_{k=1}^{K} f_{kj}$		$\sum_{i=1}^{A} a_{iC} + \sum_{k=1}^{K} f_{kC}$
Total overlay	$\frac{1}{2}\sum_{j=1}^{C} \left\| \sum_{k=1}^{K} f_{kj} \right\|$				

The vector **r** encompasses adjusted expected returns of assets and currencies, calculated per equation (5) through subtracting expected interest rates from asset returns and incorporating them into currency returns. The conditional value-at-risk (CVaR) portfolio risk measure is employed to evaluate the actual downside risk stemming from the joint distribution modeled in Section 2.2.

The mean-CVaR portfolio optimization problem with overlay constraints is subsequently formulated as:

minimize:
$$\alpha + \frac{1}{h(1-\beta)} \sum_{d=1}^{h} u_d \qquad (7a)$$

subject to:

$$\mathbf{x}^{\mathrm{T}}\mathbf{r}_d + \alpha + u_d \geq 0, \quad (7b)$$

$$\mathbf{x} = [\boldsymbol{a}, \boldsymbol{c}]^T, \quad (7c)$$

$$\mathbf{x}^{\mathrm{T}}\mathbf{r} = \boldsymbol{\mu}, \quad (7d)$$

$$\mathbf{F} \stackrel{\text{def}}{=} \mathbf{T} \circ (\mathbf{1}^T \otimes \mathbf{q}), \quad (7e)$$

$$c_j = \sum_{i=1}^{A} a_{ij} + \sum_{k=1}^{K} f_{kj} = \mathbf{F}_{kj}, \quad (7f)$$

$$\frac{1}{2}\sum_{j=1}^{C} \left|\sum_{k=1}^{K} f_{kj}\right| \leq V_u, \quad (7g)$$

$$\mathbf{1}^{\mathrm{T}}\mathbf{a} = 1, \quad (7h)$$

$$\mathbf{1}^{\mathrm{T}}\mathbf{c} = 1, \quad (7i)$$

$$u_d \geq 0, \quad (7j)$$

$$0 \leq a_{ij} \leq 1. \quad (7k)$$

The mathematical formulation of Conditional Value-at-Risk (CVaR) in an optimization problem is based on the work introduced by Rockafellar et al. (2000). In our proposed optimization problem, the linear expression for the CVaR objective is presented in (7a), and it is constrained by auxiliary variables u_d in (7b), where α and β represent the Value-at-Risk and its corresponding confidence level, respectively. It's important to note that Value-at-Risk (VaR) is computed based on an approximation of a continuous joint distribution of asset returns. To simplify the VaR computation, the actual distribution is discretized into *d* bins. Constraint (7d) specifies the target return for the portfolio. Constraints (7e) and (7f) are formulated to address overlay positions. Additionally, constraint (7g) is introduced to cap the total overlay position, preventing excessive currency risk exposure.

Steps of the Analysis

Our analysis followed a systematic process to ensure clarity and rigor:

1) Data Collection and Preprocessing – Historical data for the selected currencies and assets are collected from reliable financial databases. The data was cleaned and preprocessed to remove outliers and handle missing values.
2) Estimation of Empirical Distributions – We estimated the empirical distributions for asset and currency returns from their corresponding historical return time series. This step involves calculating the mean, variance, skewness, and kurtosis to understand the characteristics of the return distributions.
3) Construction of Joint Distributions – Using the estimated empirical distributions, we constructed joint distributions of asset returns with regular-vine copulas. This step allows us to capture the dependencies among the asset returns more accurately.

4) Scenario Generation – We generated multiple scenarios based on the joint distributions constructed in the previous step. These scenarios reflect different market conditions, including extreme events, to provide a comprehensive view of potential outcomes.
5) Portfolio Optimization under the Mean-CVaR Framework – Using the generated scenarios, we performed portfolio optimization under the mean-CVaR framework. This step involves calculating the Conditional Value-at-Risk (CVaR) for each portfolio and optimizing the asset allocation to minimize CVaR while achieving the target return.
6) Evaluation of Portfolio Performance – The optimized portfolios were evaluated through backtesting to assess their performance under historical market conditions. This step involves comparing the risk-adjusted returns of the optimized portfolios with those of traditional portfolios based on multivariate normal distributions.
7) Asset Allocation – The decision-making process involved selecting the optimal portfolio based on the evaluation results. This step normally includes considering the trade-offs between risk and return and the impact of transaction costs and hedging expenses.

In summary, this methodology provides a detailed and structured approach to multicurrency portfolio optimization using vine copulas and mean-CVaR framework. By incorporating realistic economic assumptions, rigorous estimation methods, and systematic analysis steps, we ensure the robustness and applicability of our model to real-world financial markets.

Results and Discussion

The preceding section demonstrates how to construct an international portfolio with a CVaR objective. This section describes how the portfolio was implemented using real-world data. The modelling of return distribution (whether or not nonlinear dependence of asset returns is taken into account) for CVaR calculation is the key driver for the resulting portfolios. The following experiment shows how it affects portfolio allocation and performance.

Data

In our study, the investments of interest were blue-chip stock indices, government bond indices and currencies. Our portfolio was aimed to invest in five major countries (an extension for the case of three currencies in the methodology section), i.e., the United States (US), the United Kingdom (UK), the Eurozone (EU), Japan (JP) and China (CN). These five major countries were selected due to their significant influence on global financial markets, diverse economic environments, and substantial trade volumes[2]. The base currency of the portfolio was USD, hence all FX returns were in USD.

The data was collected on a monthly basis. The in-sample period ran from January 2004 to December 2018, and the out-of-sample period ran from January 2019 to September 2022. The J.P. Morgan Markets website provides local currency returns for government bonds with maturities ranging from 1 to 10 years, while Bloomberg supplies data on currency pairs (GBPUSD, EURUSD, USDJPY, and USDCNY) as well as stock index returns for the S&P 500, FTSE 100, EURO STOXX 50, Nikkei 225, and S&P China 500.

Scenario Generation Results

In our work, we generated scenarios using two distinct approaches: RVC and MVN. The RVC method employs an R-Vine copula to depict the asset dependence structure within a return distribution that lacks a parametric form. On the other hand, the MVN method assumes that asset returns adhere to a multivariate normal distribution, with correlations delineating the dependence structure. The differing assumptions between these two methods lead to the creation of disparate scenarios.

We adopted the Monte Carlo simulation techniques outlined by Levy (2003) to generate scenarios based on a multivariate normal distribution (MVN). For generating RVC scenarios, we followed the methodology outlined in Section 2.2. Our approach encompassed five bivariate copula families: Gaussian, Student's t, Clayton, Gumbel, and Frank, along with rotated versions of Clayton and Gumbel copulas (at 90/180/270 degrees). This selection aimed to encompass a broader range of asset dependence structures. Further details on these bivariate copula families can be found in Joe (1997) and Nelsen (2007). In both scenario generation methods, we assumed equal weight of bonds and equities in each joint distribution. This assumption allowed us to standardize the analysis and focus on the impact of different dependence structures and distribution shapes.

An example of generated scenarios from RVC and MVN approaches are illustrated in Figure 1. Given MVN's assumption of normality, the return distribution shapes are symmetric, indicating equal frequencies of downside and upside events. In contrast, RVC generated samples exhibit asymmetric distributions that retain unbiased information from the raw data. Consequently, RVC distributions tend to have more outliers and an uneven distribution of downside and upside events. These distinct characteristics of scenarios generated by the two methods serve as primary factors influencing portfolio allocations in subsequent analyses across various asset classes.

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[2] The corresponding five major currencies of the country selected are the composition of IMF Special Drawing Right (SDR) basket which underscores their global economic significance and stability. More details of the SDR on https://www.imf.org/external/np/fin/data/param_rms_mth.aspx.

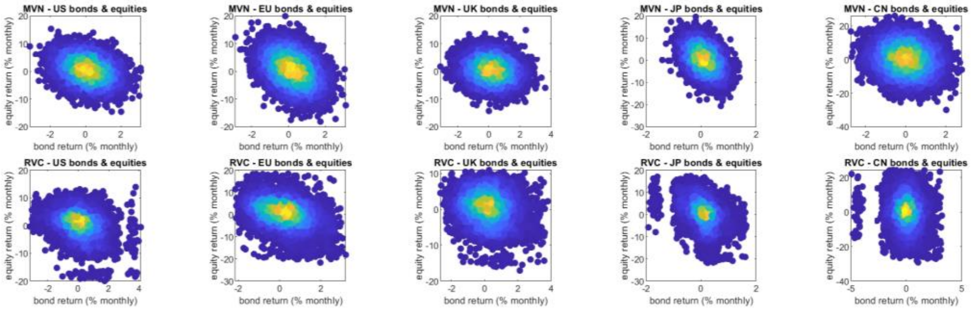

Figure 1 A comparison of bond and equity simulated returns generated with multivariate normal distribution (MVN) and R-Vine copula (RVC) for all countries. It is noticed that the RVC joint return distributions contain more extreme observations and their dependencies are asymmetric. It is assumed that bonds and equities are of equal weight in each joint distribution.

Source: Author's calculation.

Experimental Studies

Efficient Portfolios from Different Scenario Generation Methods.

Our study emphasizes the impact of varied assumptions on scenario generation. One approach assumes that return distributions of securities follow a normal distribution, with asset co-movements depicted through correlations (or linear associations). In contrast, another approach eschews assumptions about distribution families, instead of utilizing empirical distributions from historical data. The interplay between assets and currencies is captured using copulas. Consequently, the solutions derived from these two types of scenarios differ in terms of assumptions about return distribution shapes and the presence of linear or non-linear relationships between securities.

To ensure the study represents a research analysis, we have carefully chosen these methods to illustrate the impact of different assumptions on portfolio optimization. By comparing the RVC and MVN approaches, we provide a detailed analysis of how varying dependence structures and distributional assumptions affect portfolio performance. This comparison is not merely conceptual but is grounded in rigorous statistical analysis, offering valuable insights into the robustness of the portfolio under different market conditions.

Efficient portfolios are inherently most effective when assessed within the specific context in which they were formulated. This context, termed as the "environment," encompasses the returns generated under varying underlying assumptions. For instance, assuming the return distribution of an asset to be skewed and fat-tailed rather than Gaussian can lead to stark differences in the asset's return and risk profiles. Consequently, portfolios optimized based on the multivariate normal return distribution (MVN) may not be optimal in return scenarios generated using other methods, such as the regular-vine copula based scenario (RVC). The efficient frontiers depicted in Figure 2 are derived by applying optimal allocations to returns generated by scenario generators. Naturally, portfolios optimized within one environment may prove inefficient when tested within another. Different assumptions regarding return distributions significantly impact portfolio allocations and overall performance.

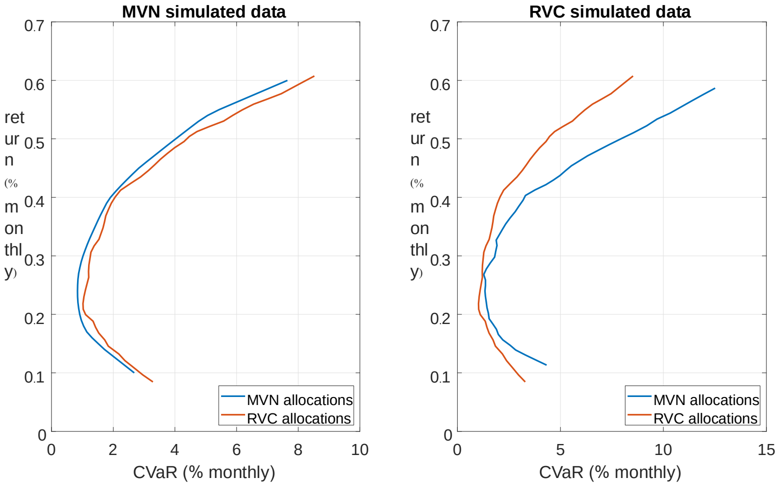

Figure 2 The comparison of efficient portfolios under different environments. RVC portfolios are deemed optimal in scenarios where assumptions about return distributions do not lean towards normality. Conversely, MVN portfolios are considered optimal when normal distribution assumptions are upheld. The left panel of the comparison graph displays the risk-return characteristics of efficient portfolios assuming normal return distributions, while the right panel depicts the risk-return profiles of the same portfolios under non-Gaussian return distributions. As expected, portfolios optimized within one environment exhibit reduced efficiency when evaluated within alternative environments.

Source: Author's calculation.

Consequently, we analyze optimal allocations produced by two scenario generation methods to determine if varying assumptions lead to differences in optimal allocations. The left panel in Figure 3 displays equity allocations, while the right panel displays foreign currency (non-USD) exposure in portfolios. Typically, portfolios with substantial equity holdings and foreign currency exposure are considered risky. Interestingly, optimal portfolios derived from both methods exhibit similar equity proportions, suggesting that departures from the normality assumption have minimal influence on bond-equity allocations.

Figure 3. The distribution of equity across portfolios is depicted in the left panel, while the right panel illustrates the extent of currency hedging through the currency overlay.

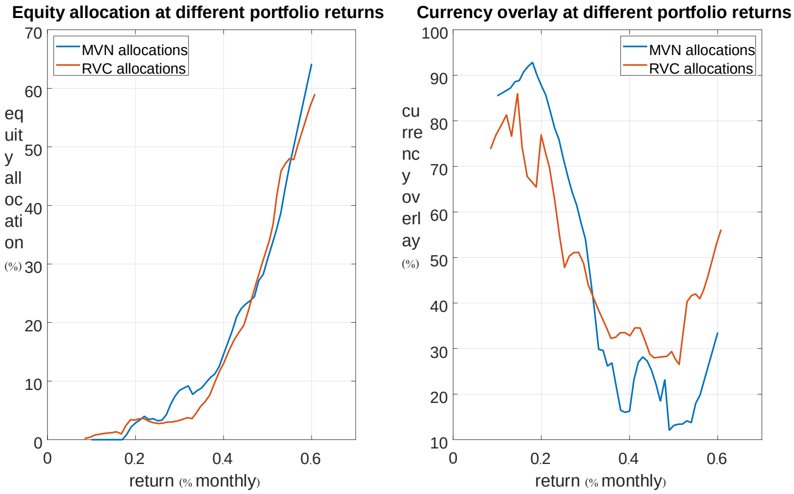

Figure 3 The distribution of equity across portfolios is depicted in the left panel, while the right panel illustrates the extent of currency hedging through the currency overlay.
Source: Author's calculation.

Robust statistical techniques have been employed to evaluate the performance of the generated scenarios. Utilizing Monte Carlo simulations, we generate a large number of potential outcomes, ensuring the reliability of our results. By comparing the RVC and MVN scenarios, we can statistically assess the impact of different assumptions on portfolio risk and return. This approach provides a comprehensive understanding of how each method influences the final portfolio allocations.

In terms of hedging against exchange rate risk, the MVN and RVC portfolios exhibit notable differences compared to their equity allocations. RVC portfolios consistently maintain a minimum 25% hedge against currency risk, whereas MVN portfolios hedge for less than 20%, particularly in situations of heightened risk appetite. This divergence highlights differing viewpoints on risk and interdependence across varying assumptions. Given that the normality assumption may not adequately account for tail risk and tail dependence, there's a potential for underestimation of risk stemming from extreme events, particularly in foreign exchange rate movements.

Portfolio Performance.

During the out-of-sample period spanning from January 2019 to September 2022, this study analyzed the cumulative returns of optimal portfolios created using two scenario generation methods. The cumulative return index, depicted in Figure 4, illustrates the compounded returns starting from an initial wealth of $100 in December 2018. To capture various risk preferences, we select three distinct optimal portfolios from the efficient frontiers based on target returns: high, medium, and low.

Table 4 provides descriptive statistics to cumulative returns in Figure 4. Notably, RVC portfolios outperform MVN portfolios across all metrics except for minimum return. This consistent outperformance of RVC portfolios is observed across different risk appetite levels. The superior performance of RVC portfolio construction could be attributed to differing perspectives on currency hedging. Figure 5 illustrates the mean returns and CVaRs of assets, along with the FX return in USD (adjusted for the cost of carry as explained in Section 2.2.2).

The data illustrates that US, EU, and UK equities offer superior returns with comparatively lower risk than other options. In portfolios seeking higher risk tolerance, there's a necessity to boost equity allocations, particularly focusing on US, EU, and UK markets. However, expanding exposure to equities from these regions also introduces FX risk, notably with EUR and GBP being among the riskier currencies. The optimal strategy revolves around securing robust equity returns while minimizing FX risks, achievable through strategic currency overlay. The RVC portfolios, with their heightened FX hedging activities as depicted in Figure 3, demonstrate superior performance as a result.

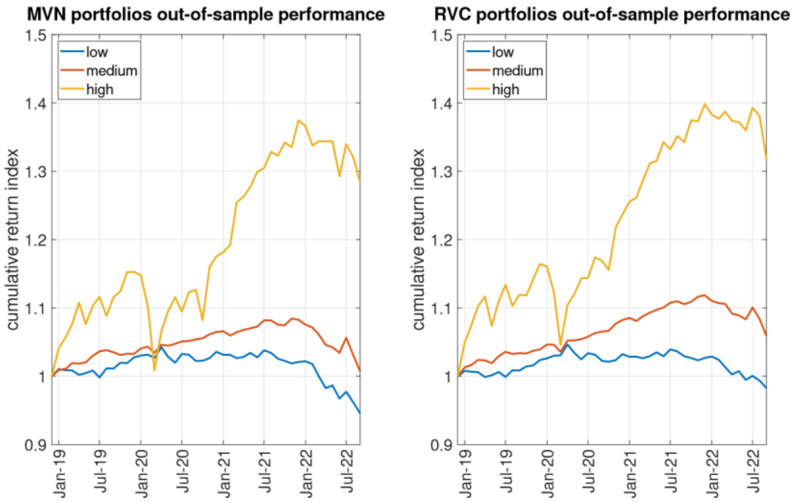

Figure 4 Cumulative wealth over the out-of-sample period (January 2019 to September 2022) of optimal portfolios from two scenario generation methods; RVC and MVN approaches. Each curve represents the cumulative wealth trajectory of optimal portfolios with high, medium, and low target returns.

Source: Author's calculation.

Table 4 The descriptive statistics present the out-of-sample performances of MVN and RVC portfolios. RVC portfolios exhibit superior risk and return attributes across various risk appetite levels.

Statistic	MVN			RVC		
	low	medium	high	low	medium	high
mean (%)	-0.04	0.13	0.64	-0.12	0.02	0.60
std dev (%)	0.67	0.73	2.41	0.87	0.81	2.80
min (%)	-1.28	-2.31	-6.82	-1.96	-2.39	-8.40
max (%)	1.54	1.61	5.60	1.54	2.14	7.23
max drawdown (%)	-6.10	-5.34	-10.18	-9.32	-7.13	-12.51

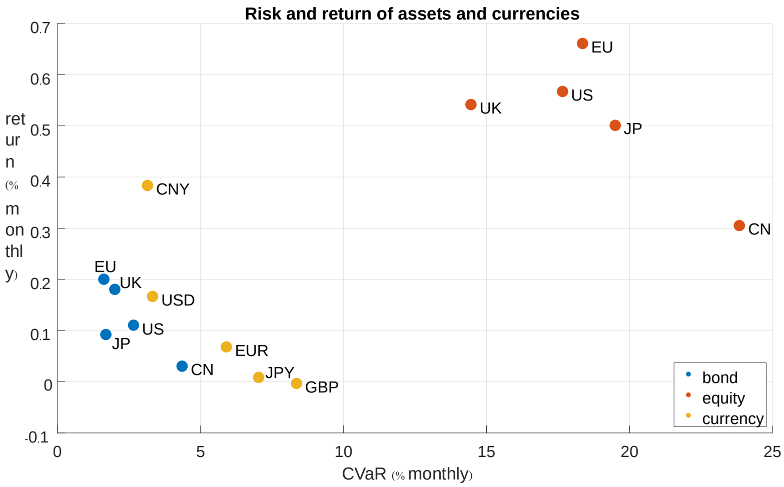

Figure 5 The risk (measured by CVaR) and return (historical average from in-sample data) profiles of assets and currencies (FX return in USD) are considered for portfolio construction. These risk and return calculations are adjusted for the cost of carry.
Source: Author’s calculation.

Conclusion

This study introduces a mean-CVaR optimization model for international portfolios with currency overlay, effectively managing currency exposure through the integration of foreign exchange forwards. This dual approach offers flexibility for both hedging and speculating on currency risks. By optimizing asset allocation and forward positions simultaneously, the model seeks to achieve optimal exposure in both asset and currency realms.

To create realistic scenarios for CVaR computation, we employ the regular-vine copula (RVC) to model nonlinear dependencies in asset returns. This methodology addresses the complexities of a multicurrency portfolio with a non-normal return distribution. We then compare the characteristics and performances of optimal portfolios derived from the RVC method with those optimized under the assumption of a multivariate normal return distribution (MVN).

Our findings reveal significant differences between RVC and MVN optimal allocations, especially in FX hedging positions (currency overlay). RVC portfolios generally exhibit higher levels of currency overlay across various risk levels due to their ability to capture extreme events more comprehensively through the regular vine copula model. Out-of-sample performance evaluations show that RVC portfolios outperform MVN portfolios in both risk and return metrics. These results highlight the importance of modeling scenarios with non-normal return distributions and nonlinear dependence structures for portfolio optimization. The superior performance of RVC portfolios underscores the value of incorporating copula methods to better manage and mitigate risks in international investments.

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