Simulated Credit VaR

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This paper attempts to examine and analyze the effects of variations in different risk factors - namely credit quality, portfolio diversification/concentration in terms of number of obligors, portfolio diversification/concentration in terms of correlation between obligors, and recovery rate - to a credit portfolio risk in the context of Value-at-Risk (VaR). Simulation results depict that the 99-percentile credit VaR exponentially increases as the portfolio credit quality declines. The phenomenon also occurs with loans of lower seniority. Meanwhile, the portfolio credit VaR decreases as the number of obligors in the credit portfolio increases, whereas, as the correlation between obligors declines, the portfolio credit VaR is accordingly lower. In addition, we find that for some particular credit portfolios, the VaR estimates are unusual. Such estimates should be carefully observed and investigated. Otherwise misleading information may lead to adverse consequences. The results of the study are useful for risk managers and/or practitioners in forming a credit portfolio and assessing credit portfolio risk factors.

1. Introduction

The recent turmoil in the capital market in 1997 and 1998 has highlighted the need for risk assessment of financial institutions' portfolios, including both their trading and lending books. It is necessary to keep risk within a certain level in relation to capital, considering that financial institutions must control their risk to maintain the safety and soundness of their operations. As a result, many techniques for monitoring both market risk and credit risk have been widely adopted by practitioners.

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Credit risk, as a major type of risk to which most financial institutions are exposed, has become a key risk management challenge. Globally, institutions are taking on an increasing amount of credit risk. As credit exposures have multiplied, the need for sophisticated risk management techniques for credit risk has also increased.

In the past, most financial institutions managed credit risk by more rigorous enforcement of traditional credit processes such as stringent underwriting standards, limit enforcement, and counterparty monitoring. Such processes were often incorporated at the management's judgement, which is sometimes considered arbitrary. As a result, nowadays, more practitioners are seeking to quantify and integrate risk assessment by using an approach that enables them to capture exposure to risk factors affecting their portfolio credit risk.

The demand is partly enhanced by the new Basel Accord - Consultative Document, which is newly distributed to the world banking sector for comment and which pays more attention to banks' exposure to credit risk, market risk, and operational risk, and takes these risk exposures into account for the required capital adequacy calculation. One major change in the new Accord is that financial institutions are allowed to use their own internal models in their portfolio credit risk estimation. Therefore, an accurate quantification of credit risk will result in an appropriate portion of capital being reserved.

To quantify risk, many techniques have been initiated. Value-at-Risk (VaR), a new advanced approach, is one of the techniques currently being widely used as a tool to manage portfolio risk among practitioners.

Concept of Value-at-Risk

Value-at-Risk (VaR) is one of the latest advancements in risk measurement. It is defined as the maximum potential negative change in value of a portfolio with a specified percentile of the probability distribution over a given time horizon. Therefore, the specified percentile is usually at the lower end of the distribution. For example, a VaR estimate of 5% with a 99% confidence interval over a one-day risk horizon suggests that there is a 1% chance that the portfolio value may drop by more than 5% of its current value.

Analytically, VaR can be formulated as follows:

$$\Pr [W_{t+h} - W_t < -VaR_w(h)] = a$$

where W_t is the portfolio value at time t and $VaR_w(h)$ is the VaR value of the portfolio W with a given degree of confidence of a, over a time horizon h.

The confidence level (1-a) is typically chosen to be at least 95% and is often as much as 99% or more. The time horizon, h, varies with the estimate of VaR made by management and with asset liquidity.

This potential loss value is useful to any business organization. It reflects the strength and vulnerability of the organization. It is also especially important for the banking sector, where financial soundness is crucial. Evidently, many bank regulators are now introducing a framework to specify bank capital requirements based on Value-at-Risk.

Although the definition of Value-at-Risk is common, there is a great variety of VaR methodologies. These methodologies differ in the details of how changes in values of assets are estimated as a result of market movements, and may be classified into two major approaches: analytical approaches and simulation approaches.

Analytical approaches are methods of VaR estimation that are based on statistical principles. They assume that asset returns follow a particular distribution and, in principle, derive a risk measure from expected variance over the next time period.

The simplest algebraic expression for calculating VaR can be written as:

$$VaR = (VRV^t)^{1/2}$$

where

$V = [P_1 x \propto x\, \sigma_1, P_2 x \propto x\, \sigma_2, \ldots\ldots, P_n x \propto x\, \sigma_n]$

P = Value of assets 1 to n

R = Correlation matrix of n asset returns

$\propto$ = Multiple of standard deviation to determine the confidence interval

σ = Volatility of asset returns 1 to n

This calculation is based on the assumption that expected returns are normally distributed with a mean of zero.

Analytical approaches can effectively estimate the risk of linear positions. Moreover, they require less time and cost. When these approaches incorporate second-order or even higher-order derivatives, Delta Approximation or Delta-Gamma Approximation, they can be extended to account for non-linear positions.

The second set of approaches, typically referred to as simulation approaches or Full Valuation models, rely on revaluing a portfolio of instruments under different scenarios. How these scenarios are generated differs and depends on the method applied, which can be either Monte Carlo simulation or historical simulation. These models normally provide a richer set of risk measures since the entire distribution of returns is also generated together with the VaR estimate.

Comparing the two simulation approaches, Monte Carlo VaR randomly generates scenarios based upon some assumed joint probability distribution of risk factors. Historical market data is used to infer statistical characteristics such as volatilities and correlations for the assumed distribution. Like Monte Carlo VaR, historical VaR must somehow select a set of scenarios to reflect the unknown distribution of risk factors. In contrast to Monte Carlo VaR, historical VaR draws scenarios directly from historical data.

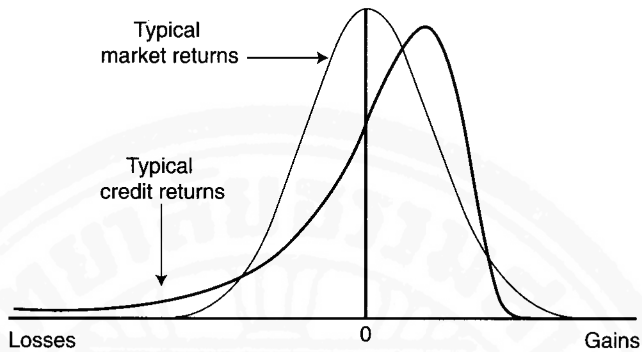
Holton (1998) concluded that Monte Carlo VaR has some advantages over historical VaR in that its convergence error can be made as small as desired by using a large number of scenarios. Moreover, because it bases volatility and correlation estimates on the most recent data, Monte Carlo VaR is more reflective of current market conditions. On the other hand, because historical VaR draws scenarios from actual market data, the scenarios reflect subtleties in market behavior that Monte Carlo VaR would miss. Additionally, no assumption regarding the distribution of asset returns is required for historical simulation.

Nonetheless, simulation approaches are computationally intensive and time-consuming, especially when they involve large portfolios under a significant number of scenarios.

Nature of Credit Risk

Modeling portfolio risk in credit portfolios is neither analytically nor practically easy. Fundamental differences between credit risks and equity price risks make equity portfolio theory problematic when applied to credit portfolios. Equity returns are relatively symmetric and well approximated by normal distributions. Thus, the two statistical measures, mean and standard deviation of portfolio value, are sufficient to assess the market risk and quantify percentile levels for equity portfolios. In contrast, credit returns are highly skewed and fat-tailed. Therefore, we need more than mean and standard deviation to assess the risk of credit portfolios accurately.

Figure 1. Loss Distribution



The long lower tail of the distribution of credit returns is caused by defaults, indicating a chance of a fairly large amount of loss. Meanwhile, credit return has proven to have a small amount of gain through its net interest earnings.

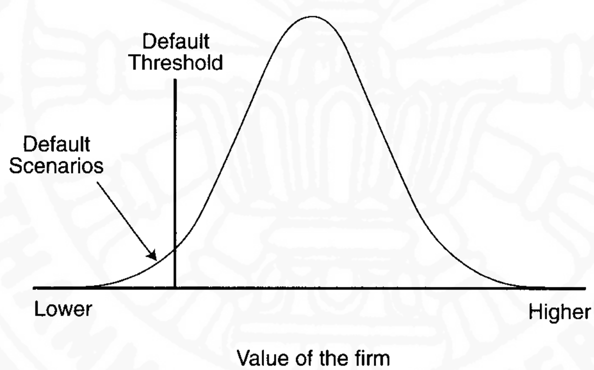
An assessment of credit risk should be performed in the context of a portfolio. The primary reason to take a quantitative portfolio approach is so that the concentration risk, additional portfolio risk resulting from increased exposure to one obligor or groups of correlated obligors, can be systematically addressed. Another important reason to take a portfolio view of credit risk is to more rationally and accountably address portfolio diversification.

Portfolio credit risk is also driven by other risk factors, namely probability of credit quality migration, default probability, and recovery rate.

Credit quality migration is the change in creditability of a counterparty from one rating class to another, including default, within a time horizon. This will result in a change in market value of the credit instrument and eventually a change in the loss distribution of a credit portfolio.

Default risk can be defined as uncertainty as to whether obligors will fail to service debt as pre-determined with creditors. Default risk varies across the credit quality of the obligors. Figure 2 illustrates a framework for thinking about default as a function of the underlying value of the firm. The framework was proposed by Robert Merton (1974), who stated that the credit risk component of a firm's debt can be valued like a put option on the value of the underlying assets of the firm, which is random with some distribution. (Here, we assume the normal distribution of the asset value.) If the value of assets falls below the amount of outstanding liabilities, which is referred to as the default threshold, it will be impossible for the firm to satisfy its obligations.

Figure 2. Default Threshold



Recovery rate reflects the probability that the defaulted obligors can recover and service their debt obligations. The amount of residual value of a credit exposure mainly depends on this factor.

Comparison of Current Credit Risk Models

During the late 1990s, a number of initiatives of credit risk models were made public. Described in a study by Crouhy, Galai, and Mark (2000), CreditMetrics from JP Morgan is based on credit migration analysis, i.e. the probability of moving from one credit quality to another, including default, within a given time horizon, which is often taken arbitrarily to be one year.

CreditMetrics models the full forward distribution of the values of any bond or loan portfolio, where the changes in values are related to credit migration only, while interest rates are assumed to evolve in a deterministic fashion. The credit VaR of a portfolio is then derived in a similar fashion as for market risk. It is simply the percentile of the distribution corresponding to the desired confidence level.

Meanwhile, KMV Corporation has developed a credit risk methodology, as well as an extensive database, to assess default probabilities and the loss distribution related to both default and migration risks. KMV's-methodology differs somewhat from CreditMetrics as it relies upon the "Expected Default Frequency", or EDF, for each issuer, rather than upon the average historical transition frequencies produced by the rating agencies for each credit class.

Both approaches, CreditMetrics and KMV, rely on the asset value model originally proposed by Merton (1974), but they differ quite substantially in the simplifying assumptions they require in order to facilitate the implementation of the model.

CreditRisk+, devised by Credit Suisse Financial Products (CSFP), only focuses on default. This model assumes that default for individual bonds or loans follows a Poisson process. Credit migration risk is not explicitly modeled in the analysis. Instead, CreditRisk+ allows for stochastic default rates which partially, although not rigorously, account for migration risk.

Finally, like CreditRisk+, CreditPortfolioView, initiated by McKinsey, measures only default risk. It is a discrete time multi-period model, where default probabilities are a function of macro-variables which drive credit cycles, such as unemployment, the level of interest rates, the growth rate in the economy, government expenses, and foreign exchange rates.

2. Methodology

In this paper, credit risk is defined as the risk arising from the failure of obligors to serve their debt obligations in a proper manner. Losses are then incurred in the credit portfolio. I attempt to study the effects of variation in default probability, default correlation, number of obligors, and recovery rate, which are considered to be the key risk factors of a credit portfolio according to the current credit risk models proposed in the market. The fundamental framework of JP Morgan's-CreditMetrics is used as the reference, and the Monte Carlo simulation is employed as a tool to generate scenarios under the framework.

To make the scope of the study precise and clearly reflect the effect of the factors being examined, hypothetical credit portfolios are constructed and the corresponding VaR is calculated under the following assumptions:

(1) The VaR of the credit portfolio is measured at the 99% confidence level.

(2) A risk horizon of one year is predetermined. This is in accordance with the general practice of many financial institutions of reviewing risk management guidelines and policies on an annual basis. Therefore, only the VaR at the end of the horizon is considered.

(3) Only default and non-default events are considered in measuring the VaR of the credit portfolio. The model in this study focuses on capturing the maximum potential loss, given a certain confidence level and risk horizon, of illiquid debt instrument. Revaluation at the end of the horizon of non-default risky debt, by incorporating the effects from transition of credit quality, will not be implemented.

(4) To define default and non-default events, I rely on the asset value model in the process of scenario generation. In addition, to standardize the results, each obligor's asset returns are assumed to have a mean of zero and a standard deviation of one.

(5) The integration of market risk is left for further study; therefore,

interest rate uncertainty and the present value of expected future cash flow are not taken into consideration.

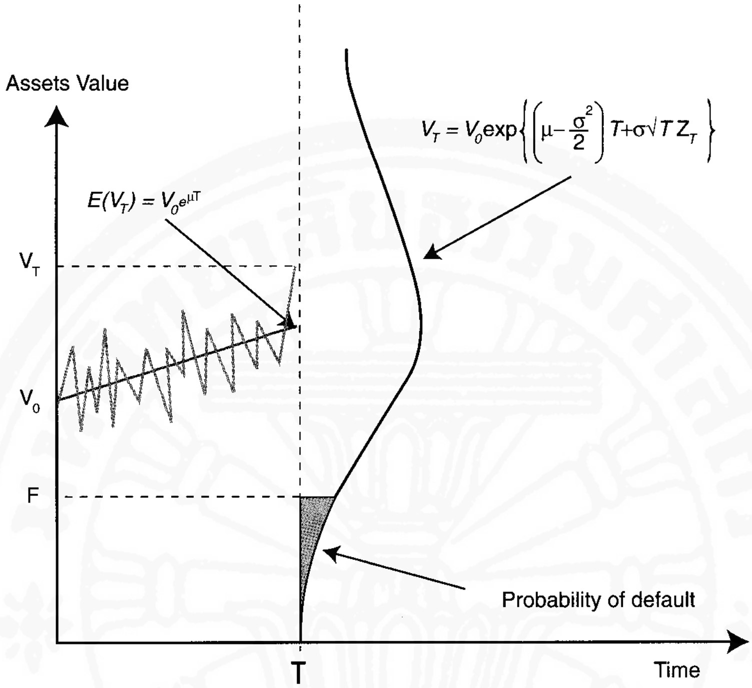
(6) The effects of collateral and loan loss reserve are presumed to be totally reflected by the recovery rate predetermined for each hypothetical portfolio.

From a hypothetical portfolio whose key risk factors are predetermined—default probability, correlation between obligors, number of obligors in the portfolio, and recovery rate—scenarios of credit quality transition, including default, for each obligor in the next one year are generated through simulation.

With respect to each obligor's credit quality transition, I take into account the correlation between individual obligors by decomposing the default correlation matrix and incorporating it into the generated random numbers. By doing this, simulation in consideration of chain-default is possible, which enables us to take into consideration diversification effects in a portfolio context.

Given a perfect market where all transaction costs, taxes, and informational differences are free, the value of the firm will be independent of its capital structure and is simply the sum of debt and equity values. According to Merton's asset value model, provided all the firm's assets can be converted into cash at the maturity date T without any liquidity constraints or transaction costs, if the terminal value of the firm's assets is greater than the value of its debt obligation, all of the firm's debt will be fully paid off. Otherwise, the firm defaults. The diagram below presents a summary of this theoretical framework.

Figure 3. Asset Value Model



V_0	=	Value of assets at time 0
V_T	=	Value of assets at time T
F	=	Face value of debt
T	=	Time to maturity of debt
Z	=	Standard Weiner process

For the non-default scenario, no expected loss is to be estimated. For the default event, the loss amount that may be incurred for each scenario is calculated and then summed up to be the portfolio loss. The process is repeated 10,000 times and then the distribution of the results is measured. As a consequence, portfolio credit VaR is achieved. I then investigate the effects of variations in the key risk factors for each hypothetical portfolio.

As the base case scenario, the four interest risk factors under consideration are determined to be as follows:

Table 1. Base Case

Portfolio Value	Default Probability	No. of Obligors in Credit Portfolio	Default Correlation between Obligors	Recovery Rate	Recovery Std.
100 mill.	0.18%	100	0	53.8%	26.86%

Note that, to fairly reflect the effects of variation in each risk factor on the credit portfolio:

(1) The total portfolio value of Bt100 million is allocated among obligors equally.

(2) All the obligors in a credit portfolio are assumed to have the same default probability, recovery rate, and default correlation between obligors in each simulation.

To simulate scenarios reflecting the effects of variations in each key risk factor on a credit portfolio, I classified the simulation process into four classifications. In each classification, the risk factor being examined is varied in the range of interest, while the other factors are fixed as determined in the base case scenario. The effect of increasing the number of obligors in a credit portfolio is studied based on the variation determined in case I. The effects of variations in the default correlation between each obligor, in the recovery rate, and finally in the default probability are simulated in case II, case III, and case IV, respectively.

Case I

Table 2. Study of the effect of variations in the number of obligors

Variation	Default Probability	No. of Obligors in Credit Portfolio	Default Correlation between Obligors	Recovery Rate	Recovery Std.
1	0.18%	10	0	53.8%	26.86%
2	0.18%	25	0	53.8%	26.86%
3	0.18%	50	0	53.8%	26.86%
4	0.18%	75	0	53.8%	26.86%
5	0.18%	100	0	53.8%	26.86%
6	0.18%	200	0	53.8%	26.86%
7	0.18%	500	0	53.8%	26.86%

Case II

Table 3. Study of the effect of variations in the default correlation

Variation	Default Probability	No. of Obligors in Credit Portfolio	Default Correlation between Obligors	Recovery Rate	Recovery Std.
1	0.18%	100	0	53.8%	26.86%
2	0.18%	100	0.25	53.8%	26.86%
3	0.18%	100	0.50	53.8%	26.86%
4	0.18%	100	0.75	53.8%	26.86%
5	0.18%	100	0.9	53.8%	26.86%

Case III

Table 4. Study of the effect of variations in the recovery rate

Variation	Default Probability	No. of Obligors in Credit Portfolio	Default Correlation between Obligors	Recovery Rate	Recovery Std.
1	0.18%	100	0	53.80%	26.86%
2	0.18%	100	0	51.13%	25.45%
3	0.18%	100	0	38.52%	23.81%
4	0.18%	100	0	32.74%	20.18%
5	0.18%	100	0	17.09 %	10.90%

Table 5. Recovery rate classified by seniority

Seniority	Recovery Rate	Recovery Std.
Senior Secured	53.80%	26.86%
Senior Unsecured	51.13%	25.45%
Senior Subordinated	38.52%	23.81%
Subordinated	32.74%	20.18%
Junior Subordinated	17.09 %	10.90%

Case IV

Table 6. Study of the effect of variations in the default probability

Variation	Default Probability	No. of Obligors in Credit Portfolio	Default Correlation between Obligors	Recovery Rate	Recovery Std.
1	0.02%	100	0	53.80%	26.86%
2	0.04%	100	0	53.80%	26.86%
3	0.06%	100	0	53.80%	26.86%
4	0.18%	100	0	53.80%	26.86%
5	1.06%	100	0	53.80%	26.86%
6	5.20%	100	0	53.80%	26.86%
7	19.79%	100	0	53.80%	26.86%
8	50.00%	100	0	53.80%	26.86%

Variations in the default probability are intentionally applied in order to study the effects of variations in credit quality, reflected by the credit ratings of international rating agencies, to the value of the credit portfolio. Each of the default probabilities can be related to the international rating standard as follows:

Table 7. Default probability corresponding to credit rating

Rating	Default Probability
AAA	0.02%
AA	0.04%
A	0.06%
BBB	0.18%
BB	1.06%
B	5.20%
CCC	19.79%

Source: Standard & Poor's

In the case that an obligor's rating is below the investment rate, with a considerably high chance of default, a 50% probability of default is applied.

Therefore, it is obvious from table 1 that I assume in the base case scenario an extension of secured senior debts to 100 obligors with a loan amount of Bt1 million each. The credit worthiness of all obligors is equivalent to the investment-rate debt instrument or, in other words, all the obligors are rated "BBB" by Standard & Poor's.

3. Results and Implications

After many simulations were repeated with respect to the variations in each of the factors under consideration, the results were compared and analyzed. This consists of two parts. The first part, Experimental Results, presents the results from the simulations. The implications for risk managers, credit portfolio managers, and other practitioners are discussed in the second part.

Experimental Results

Effects of diversification contributed by increasing the number of obligors

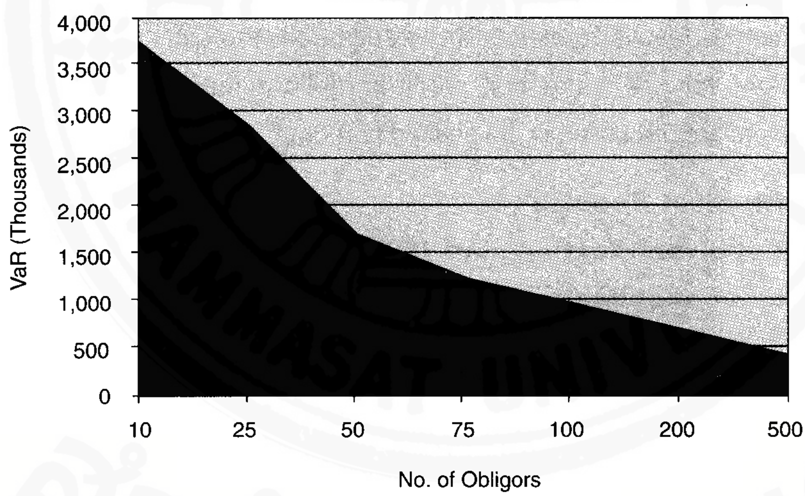
Referring to portfolio theory for investment, as the number of securities increases, given the same level of risk for each security, the portfolio risk decreases. Does the risk of the credit portfolio follow this theory?

To determine the answer to this question, seven hypothetical portfolios composed of 10, 25, 50, 75, 100, 200, and 500 obligors were created. The total amount of Bt100 million was allocated equally among obligors. All the obligors were rated at the investment rate for which the default probability is 0.18%. I assumed the secured-seniority of all debts. Then, simulations of all hypothetical portfolios were generated. The figures below present the credit VaR estimates resulting from the simulations.

Table 8. VaR estimates from variations in the number of obligors

No. of Obligors	99 Percentile Credit VaR
10	3,760,704
25	2,858,259
50	1,682,062
75	1,223,269
100	945,581
200	668,519
500	414,156

Figure 4. VaR estimates from variations in the number of obligors

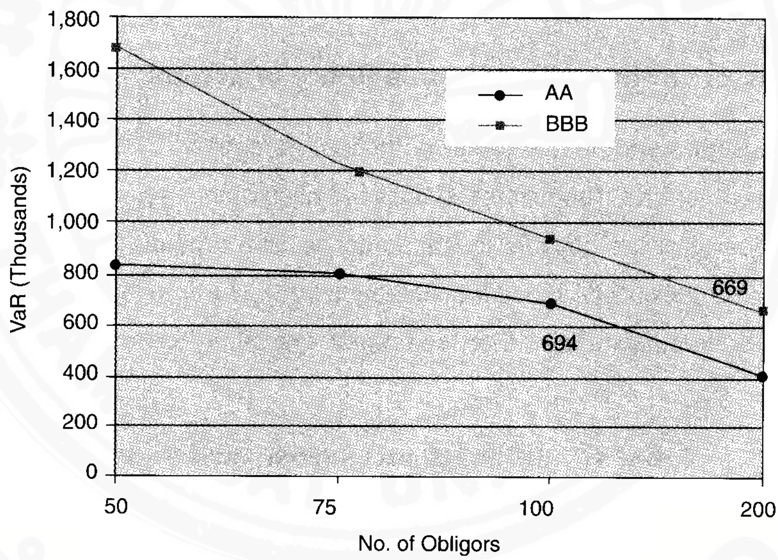


The results indicate that as the number of obligors in the hypothetical credit portfolio increases, the credit VaR at the 99% confidence level decreases at a diminishing rate. The results reflect the benefit of diversification contributed by an increasing number of obligors in the credit portfolio. I also compared the results from a hypothetical portfolio comprised of BBB-rated obligors with the results of a portfolio composed of AA-rated obligors. The results are presented below.

Table 9. Comparison of VaR estimates for AA and BBB portfolios

No. of Obligors	99 Percentile VaR	
	AA	BBB
50	835,506	1,682,062
75	801,908	1,223,269
100	693,810	945,581
200	411,944	668,519

Figure 5. Comparison of VaR estimates for AA and BBB portfolios



The figures above show that an increase in the number of obligors in the BBB-rating portfolio can help to significantly reduce the portfolio credit VaR. When the number of obligors reaches 200, the portfolio credit VaR can be reduced to a level below the credit VaR of a higher profile AA-rating portfolio containing 100 obligors.

At this point, it can be concluded that increasing the number of obligors in a credit portfolio helps reduce the portfolio credit VaR. Moreover, a credit portfolio manager can reduce the VaR of a lower-rating-profile credit portfolio by increasing the number of obligors in the portfolio until the portfolio quality, in terms of credit VaR, is equivalent to the quality of a higher-rating-profile credit portfolio. However, note that the benefit from the diversification is limited. After the number reaches a certain level, a portfolio risk manager cannot gain much more benefit by further increasing the number of obligors.

Note that the result of variations in the number of obligors is in general in line with the result of the study by Oda and Muranaga (1997). However, the results from variations in other risk factors are not comparable due to significant differences in the set of assumptions and the framework of the study.

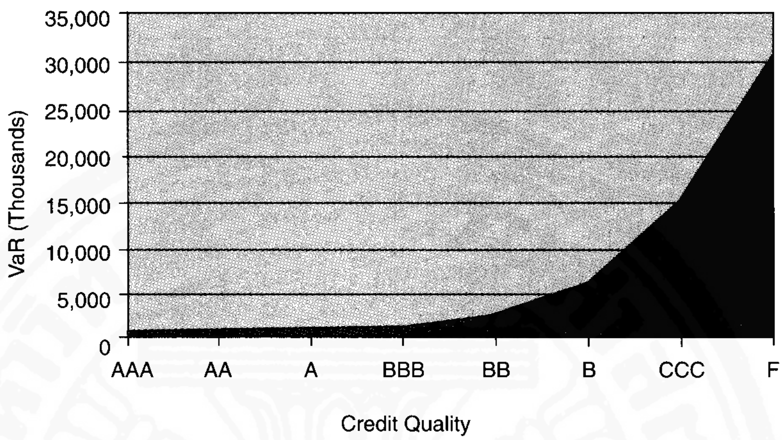
Effects of variations in the probability of default

Eight hypothetical credit portfolios, comprised of 100 obligors each, were formed. The difference between each of these portfolios is the credit quality of the obligors in the portfolios. The probability of default, reflecting credit quality, varied from 0.02% to 50%. The credit VaR resulting from the simulations was then compared and analyzed as presented below.

Table 10. VaR estimates from variations in the default probability

PD	Equivalent Rating	99 Percentile Credit VaR
0.02%	AAA	405,865
0.04%	AA	693,810
0.06%	A	790,738
0.18%	BBB	945,581
1.06%	BB	2,216,311
5.20%	B	5,783,209
19.79%	CCC	14,593,792
50.00%	-	30,514,630

Figure 6. VaR estimates from variations in the default probability



As the portfolio credit quality decreases, thus changing the default threshold, the 99 percentile credit VaR increases exponentially. As shown in the results, considering the 99 percentile credit VaR of the BBB- and B-rating portfolio, the credit risk in terms of VaR may be multiplied by approximately 600% by decreasing the credit quality by only two notches. Therefore, as a risk manager, the trade-off between a better diversification from an increase in the number of obligors and a potential radical rise in portfolio credit risk resulting from lower-rated obligors has to be thoroughly considered.

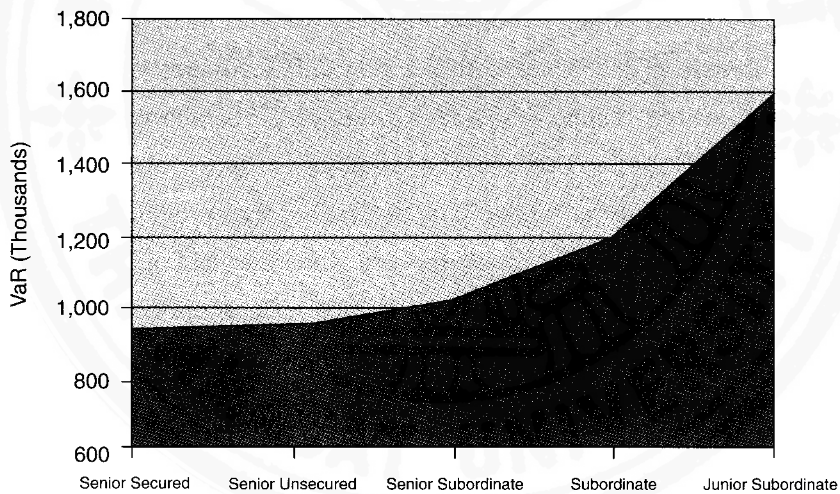
Effects of variations in the recovery rate

To study the effects of variations in recovery rate implying the different seniorities of debt instrument to a portfolio credit risk, I constructed five hypothetical credit portfolios composed of 100 BBB-rating obligors with a different seniority class for each portfolio. Each class of seniority implies a different mean and volatility of recovery rate, referring to the empirical study in the United States. The results from variations in the mean and volatility of the recovery rate to the credit portfolio VaR at a confidence level of 99% are as follows:

Table 11. VaR estimates from variations in the recovery rate

Seniority	Recovery Rate	Recovery Std.	99 Percentile VaR
Senior Secured	53.80%	26.86%	945,581
Senior Unsecured	51.13%	25.45%	953,336
Senior Subordinated	38.52%	23.81%	1,017,393
Subordinated	32.74%	20.18%	1,194,175
Junior Subordinated	17.09%	10.90%	1,578,207

Figure 7. VaR estimates from variations in the recovery rate



Changes in debt seniority do not affect a portfolio credit VaR linearly. On the contrary, the 99 percentile credit VaR exponentially increases as seniority of debt declines. Obviously, seniority of debt is one of the critical issues for portfolio credit risk management. A risk manager should take the results into account when a new loan with a lower seniority is extended. In some cases, particularly when the benefit from diversification is diminished, the benefit from increasing the number of obligors may be unreasonable considering the potential rise in the portfolio credit risk resulting from a lower-seniority debt being added to the portfolio.

Effects of diversification contributed by default correlation between obligors

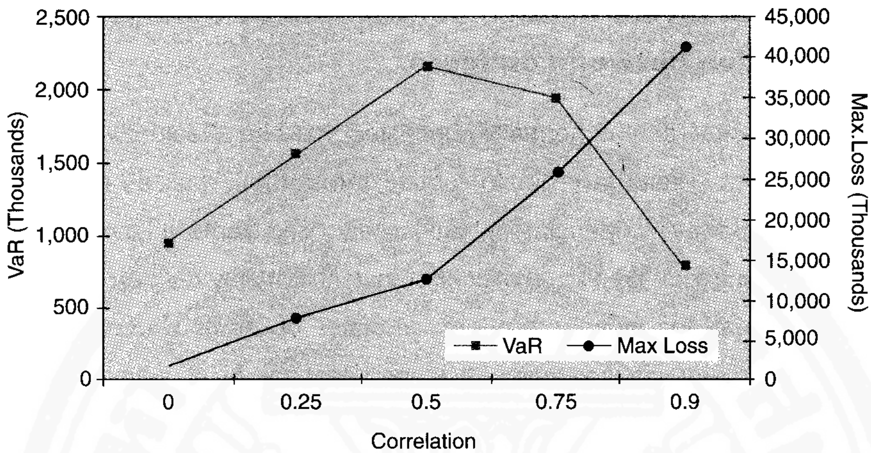
In general, it is normally suggested that one should diversify an investment portfolio in order to reduce the portfolio risk as much as possible. However, financial institutions may find some advantages in concentrating their credit portfolios. For instance, it may be more efficient to investigate and monitor the financial status of clients who specialize in a certain industry. Moreover, in some situations or economic states, business prospects may be concentrated only in a certain industry or business. As a result, concentrating the credit portfolio is inevitable.

To study the effects of diversification contributed by default correlation between obligors, I constructed five hypothetical credit portfolios, each of which contained 100 investment-rate obligors. Each loan was senior secured debt. I then varied the default correlation between obligors from one portfolio to the next. The range of variation is from 0 to 0.9. The resulting 99 percentile portfolio credit VaR estimates of the five portfolios are as follows:

Table 12. VaR estimates from variations in the default correlation 1

Credit Rating	Default Correlation	99 Percentile VaR
BBB	0	945,581
BBB	0.25	1,566,717
BBB	0.50	2,162,141
BBB	0.75	1,933,011
BBB	0.90	801,485

Figure 8. VaR estimates from variations in default correlation 1



Surprisingly, the 99 percentile portfolio credit VaR does not keep increasing as the default correlation increases, as might be expected. Instead, the 99 percentile portfolio credit VaR starts to decrease when the default correlation is higher than 0.5. Meanwhile, the maximum loss resulting from the simulation increases in accordance with a higher level of correlation.

Because of this unexpected result, a study of the effect of variations in the default correlation to other credit portfolios was conducted. I studied the effects of variations, of the same range as that applied to the BBB-rating portfolio, on AAA, AA, A, BB, B, and CCC-rating portfolios. The portfolio credit VaR resulting from the simulations can be separated into two groups. The first is the portfolio credit VaR results for credit portfolios for which the default probabilities are lower than the significance level on the lower tail, meaning AAA, AA, A, and BBB-rating portfolios. The second is the portfolio credit VaR results for credit portfolios for which the default probabilities are higher than the significance level, meaning BB, B, and CCC-rating portfolios. The results are presented in the figures below.

Table 13. VaR estimates from variations in the default correlation 2

Correlation	99 Percentile VaR			
	AAA	AA	A	BBB
0	405,865	693,810	790,738	945,581
0.25	410,327	700,503	873,038	1,566,717
0.50	280,471	645,094	859,388	2,162,141
0.75	-	98,087	557,768	1,933,011
0.90	-	-	-	801,485

Figure 9. VaR estimates from variations in the default correlation 2

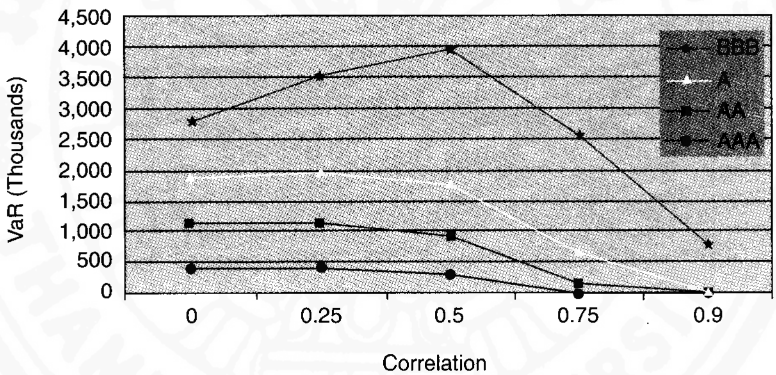
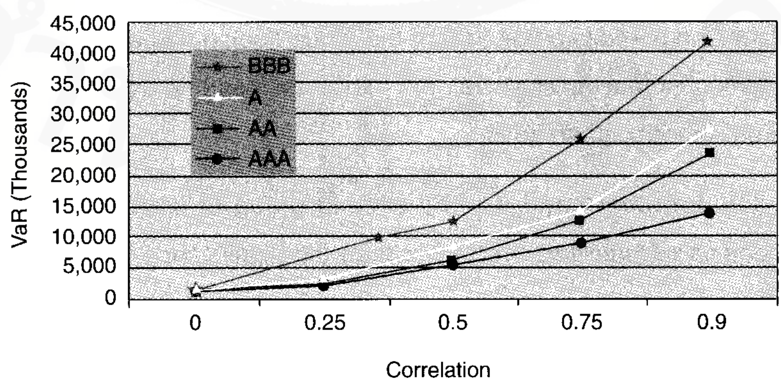
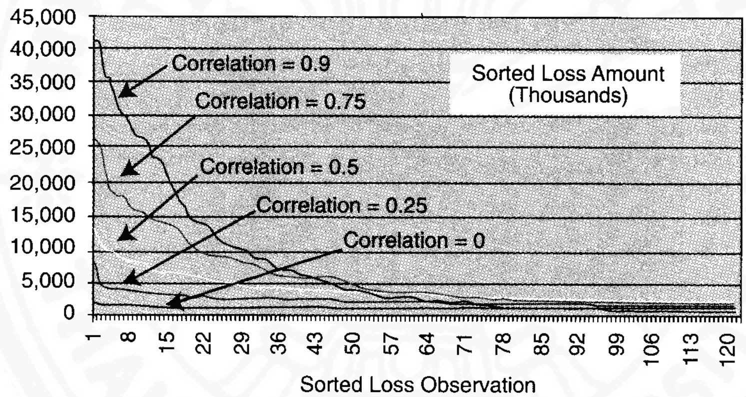


Figure 10. Maximum loss from variations in the default correlation 2



The resulting portfolio credit VaR of the first group of portfolios, whose default probabilities are lower than the significance level, does not keep increasing as the correlation rises. In contrast, it is noticeable that when the correlation increases to a certain level, the portfolio credit VaR responds negatively to a further increase in correlation. Meanwhile, the maximum losses resulting from the simulations increase in accordance with a higher level of correlation. A graphical presentation of the portfolio credit VaR of the group is provided below.

Figure 11. Sorted loss observations of low default probability portfolios



From the plot of the sorted loss amount resulting from 10,000 simulations, we can see that as the correlation increases, the sorted series of loss amount declines faster than a series with a lower correlation. As a result, the portfolio credit VaR of the group does not keep increasing as correlation increases. However, the maximum loss of a higher correlation series is more catastrophic.

On the other hand, for the second group of credit portfolios, whose default probabilities are higher than the significance level, the 99 percentile VaR increases as the correlation increases. The results are presented below.

Table 14. VaR estimates from variations in the default correlation 3

Correlation	99 Percentile VaR		
	BB	B	CCC
0	2,216,311	5,783,209	14,593,792
0.25	5,286,958	14,894,913	30,582,325
0.50	9,517,992	24,905,612	40,983,014
0.75	14,674,037	36,960,634	46,793,933
0.90	20,502,796	44,310,608	48,811,000

Figure 12. VaR estimates from variations in the default correlation 3

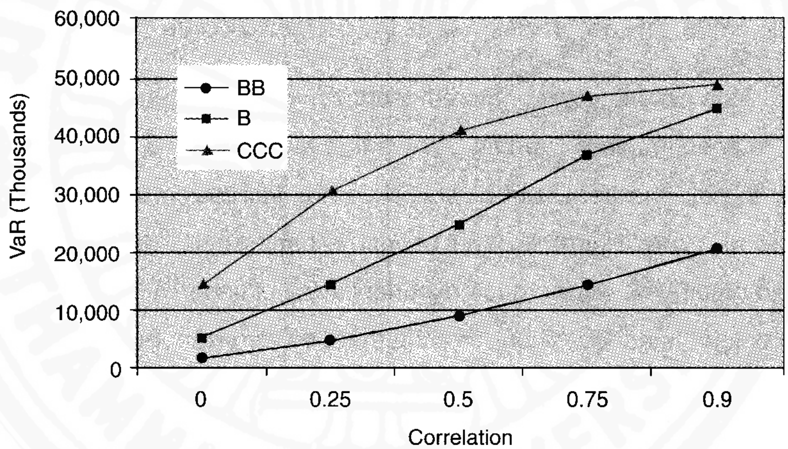
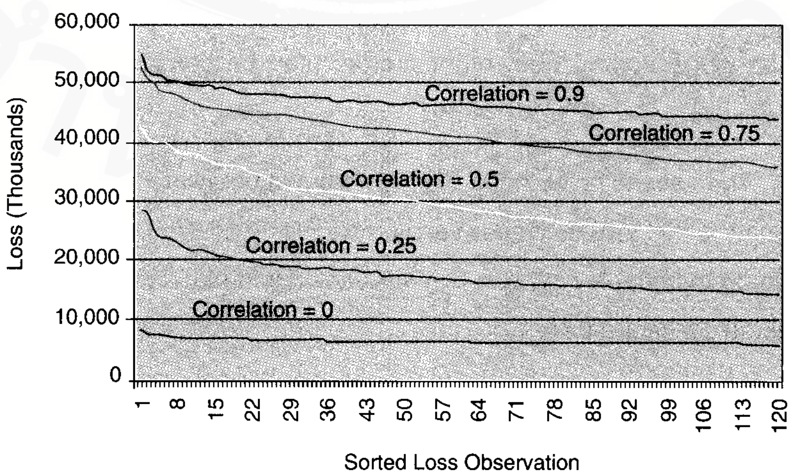


Figure 13. Sorted loss observations of high default probability portfolios



Implications for Practitioners

With the ability to quantify a portfolio credit risk, from the experimental results presented above, characteristics of hypothetical portfolio credit VaR as responses to variations in each of the key risk factors can be concluded to be as follows:

(1) The 99 percentile portfolio credit VaR decreases at a diminishing rate as the number of obligors in the portfolio increases.

(2) The 99 percentile portfolio credit VaR exponentially increases as the credit quality of the portfolio moves to a lower profile.

(3) Particularly for variations in default correlation, the VaR estimates sometimes present strange results, in which the 99 percentile portfolio credit VaR becomes lower as the correlation rises.

As a risk manager or a credit portfolio manager, there are two key points to keep in mind. The first point is that even though a risk manager is able to gain the benefit of diversification by increasing the number of obligors in a credit portfolio, the benefit is diminished. Meanwhile, the portfolio credit VaR will increase exponentially, not linearly, when a lower-profile obligor is added. Therefore, the trade-off between the benefit from diversification, which diminishes, and a potential increase in the portfolio's-risk resulting from adding a lower-profile obligor should always be kept in mind by risk managers and credit portfolio managers. A thorough investigation should be conducted for each individual case. The attempt to diversify a portfolio without considering the potential increase in portfolio risk resulting from other factors may be considered a blind diversification.

The second point that a manager should be aware of and should investigate is that VaR estimates can sometimes be unusual. Neglecting to investigate thoroughly may result in misleading information that may lead to adverse consequences.

5. Practical Applications

Attempts to quantify a portfolio credit risk and integrate the risk within an approach enabling the capture of exposure to risk factors affecting portfolio credit risk have advanced during the past few years. This advancement of studies makes a contribution to the risk management society in terms of ways to achieve an effective methodology that enables practitioners to quantifiably assess and manage the risk of a portfolio within a framework. The quantification of risk can lead to applications of corporate management, especially for financial institutions, where soundness of operations and capital adequacy are crucial.

This section describes the applicability of risk quantification to practical implementations for the risk management of financial institutions.

Determination of Capital Adequacy

After the financial crisis in 1997, concerns about the adequacy of financial institutions' capital against all the risks taken have been spilling over the banking and finance industry. Risk is related to the amount of capital a financial institution requires to achieve a sufficient level of protection against adverse circumstances. To achieve safety and soundness of operations with an adequacy of capital against risks, risk assessment has to be carried out first. Many techniques are applied in order to achieve effective comprehensive risk management. Value-at-Risk is one of the advancements widely implemented by practitioners for the assessment of risks.

Currently, lack of sufficient historical data on default events and the migration of credit quality of obligors has caused a major problem for estimation of portfolio credit risk using the Value-at-risk approach. Therefore, the Monte Carlo simulation, given a certain shape of distribution, is now considered the "at-the-best" method for measuring portfolio credit VaR. As a risk manager, understanding the effects of each risk factor on portfolio credit risk is important. A portfolio risk does affect the requirement of capital. If one can manage the risk of a portfolio effectively, the requirement of additional capital against an incremental portfolio risk will be minimized.

Risk-based Pricing and Capital Allocation

As the first application of capital adequacy determination reflects a debtholder's perspective on risk, risk is also used to adjust the return from business activities to determine whether activities are value creating or value destroying. This adjustment on return reflects a shareholder's perspective on risk. The debtholder and shareholder views of risk differ, but are related. Actions which tend to increase risk to debtholders also tend to increase risk to shareholders. This can be drawn by using the same risk measurement framework.

In order to achieve maximization of risk-adjusted returns, pricing of any business activity, particularly for credit instruments, should be risk-based pricing. The pricing should take the individual risk and the incremental portfolio risk in the case that a loan will be extended to a new client into consideration. Generally speaking, a higher-risk client should be fairly charged at a higher price relative to a lower-risk client.

Risk-adjusted return on capital, RAROC, is now widely used as a measurement of the performance of business activities. The framework measures business performance by comparing returns earned by a business unit to the capital allocated to that business unit in accordance with the risks that have been taken.

5. Conclusions

This paper proposes a framework to study the effects of risk factors (probability of default, default correlation between obligors, number of obligors, and recovery rate) on portfolio credit Value-at-Risk. The study uses the Monte Carlo Simulation technique to simulate the resulting credit VaR of different hypothetical credit portfolios. Each of the risk factors is manipulated in order to see the effects of variations in each factor on the portfolio credit VaR. The results from the simulations can be concluded to be as follows:

(1) As the number of obligors in a hypothetical credit portfolio increases, the credit VaR at a 99% confidence level decreases at a diminishing rate. The results exhibit the benefit from diversification contributed by increasing the number of obligors in the credit portfolio. The implication is that risk managers can reduce the VaR of a lower-rating-profile credit portfolio by simply increasing the number of obligors in the portfolio until the portfolio quality, in terms of credit VaR, is equivalent to that of a higher-rating-profile credit portfolio.

(2) As the portfolio credit rating gets lower (implying a higher default probability for obligors), thus changing the default threshold, the 99-percentile credit VaR increases exponentially.

(3) Changes in debt seniority do not affect a portfolio credit VaR linearly. On the contrary, the 99-percentile credit VaR exponentially increases as the seniority of debt declines.

(4) Based on the first three findings, risk managers should also be wary of blind diversification, which has a trade-off between the benefit from diversification (which is diminishing) and the potential marginal risk, which exponentially increases with the addition of a lower-profile obligor.

(5) For a portfolio with a default probability higher than the significance level, the 99-percentile portfolio credit VaR increases as the default correlation between obligors in the portfolio increases. In contrast, when the correlation in a portfolio with a default probability lower than the significance level has reached a certain point, the 99-percentile portfolio credit VaR will negatively respond to a further increase in correlation. However, the maximum losses of every portfolio increase in accordance with a rise in correlation. This shows that Value-at-Risk estimates of some portfolios can sometimes be irregular. Therefore, risk managers should be cautious and should investigate such estimates carefully. Otherwise, misleading risk information may lead to adverse consequences.

6. Recommendations for Further Study

This study was conducted under a certain set of assumptions. Further research can be conducted by changing these assumptions and examining how the results would change as a result of modifying the assumptions. For instance, instead of using a time horizon of one year, the portfolio credit VaR may be assessed by applying the actual maturity of debt instruments. It should be noted that, in all cases, a conditional credit quality migration during the period of assessment should be assigned.

It would also be interesting to collect historical data on default events of obligors in order to build up an expected default frequency and credit quality migration information for each credit class. When it is applicable, a simulation based on the true distribution can be created. The results can be compared to observe what the different effects on the portfolio credit VaR would be.

Finally, it would be challenging to extend the study to the integration of market risk, and even operational risk. With these extensions, a more complete framework of risk assessment could be implemented.

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